Mathematics Classroom Observation Protocol for Practices: Descriptors Manual

Authors

Jim Gleason
Department of Mathematics
The University of Alabama
jgleason@ua.edu

Stefanie D. Livers
Department of Childhood & Family Studies
Missouri State University
StefanieLivers@MissouriState.edu

Jeremy Zelkowski
Department of Curriculum and Instruction
The University of Alabama
jzelkowski@ua.edu

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The Mathematics Classroom Observation Protocol for Practices (MCOP^2) is a K-16 mathematics classroom instrument designed to measure the degree of alignment of the mathematics classroom with the various standards set out by the corresponding national organization that focus on conceptual understanding in the mathematics classroom including:

- Common Core State Standards in Mathematics: Standards for Mathematical Practice **(National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010)**,
- Mathematical Association of America (MAA): CUPM Curriculum Guide **(Barker, et al., 2004)**,
- American Mathematical Association of Two-Year Colleges (AMATYC): "Crossroads" **(AMATYC, 1995)** and "Beyond Crossroads" **(AMATYC, 2006)**, and
- National Council of Teachers of Mathematics (NCTM): Process Standards **(NCTM, 2000)**.

Recommended Uses

The MCOP^2 form is designed to measure the activities occurring in a mathematics classroom during a single lesson. However, if one desires to measure the overall activities of a class, the form should be used to measure at least three different class settings. **An important item to remember is that while all of the items in the observation protocol are desired qualities of a mathematics classroom, not all of them are expected to be observed during a single lesson. It is expected that this instrument be used in a formative manner on single observations. Summatively, 3-6 observations are ideal in evaluating classroom instruction.**

The MCOP^2 form is not designed to be used during a single lesson or day to evaluate the teaching and learning atmosphere of the mathematics classroom.

When completing the MCOP^2 form, it is essential that the descriptors outlined in this manual are followed to maintain the reliability of the instrument and the validity of constructs.

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 Updated: February 16th, 2018 – Appendix of Operationalized Terms

Operational Definitions of MCOP^2 Item Terminology

In December of 2017 at the Association of Public and Land-grant Universities (APLU) headquarters in Washington, DC, mathematics teacher education professionals from around the U.S. attended an MCOP^2 specialized training provided and supported by the APLU and the national Mathematics Teacher Education – Partnership (MTE-P, see link below for more information). During the event, a handful of specific terms in items on the protocol were determined that for local use, research and evaluation would improve with firmer definitions of these terms within specific items as it relates to scoring. Users of the protocol should consider these explicit definitions more specifically. Underlined terms in the appendix are those that participants aimed for enhanced clarification. In some instances, the use of the short form observation protocol may lose operational use. Therefore, it is imperative for users to be fully comfortable with the long form descriptors manual before using the short form for formal observations for research, evaluation, coaching cycles, or other formal and informal uses.

In the past two years at the Mathematics Teacher Education – Partnership annual conference and various research action cluster (RAC) meetings, it was suggested that an additional summary page be included in this descriptors manual regarding the equitable teaching practices that the development team used during the item development process. This summary appears after item 16 in the descriptors manual, and a "notes" section on page 4 of the short form has been added with this update. To keep the structure of the MCOP^2 as it was during the validity research and reliability of use unchanged, these additions are framed in the manner only in which the development team used them.

We encourage users of the MCOP^2 to consult the Appendix in this document for these operationalized terms and clarifications on more difficult items to score.

We add special thanks for Drs. Mark Ellis and Patrice Waller, California State University – Fullerton on their work during this process to improve the understanding of the MCOP^2 use in mathematics teacher preparation, evaluation, and research.

http://www.aplu.org/projects-and-initiatives/stem-education/mathematics-teacher-education-partnership/index.html

How to Score

The MCOP^2 measures two distinct factors of Teacher Facilitation and Student Engagement through two subscales of 9 items each. (The MCOP^2 is not designed to get a single score of a classroom.)

The Teacher Facilitation subscale (Cronbach alpha of 0.850) measures the role of the teacher as the one who provides structure for the lesson and guides the problem solving process and classroom discourse. To calculate the score for the Teacher Facilitation subscale, one would add the scores for items 4, 6-11, 13, and 16.

The Student Engagement subscale (Cronbach alpha of 0.897) measures the role of the student in the classroom and their engagement in the learning process. To calculate the score for the Student Engagement subscale, one would add the scores for items 1-5 and 12-15.

Item	Student Engagement	Teacher Facilitation
1	X	
2	X	
3	X	
4	X	X
5	X	
6		X
7		X
8		X
9		X
10		X
11		X
12	X	
13	X	X
14	X	
15	X	
16		X

On the short form, users will see which factor(s) the item loads. Therefore, it is critical that the observations score based on the observables of the teacher, student, or teacher & student (items 4&13).

1) **Students engaged in exploration/investigation/problem solving.**

The role of exploration, investigation, and problem solving is central in teaching mathematics as a process. In order for students to develop a flexible use of mathematics, they must be allowed to engage in exploration, investigation, and/or problem solving activities which go beyond following procedures presented by the teacher. Furthermore, problem solving can be developed as a valuable skill in itself (Barker, et al., 2004) and a way of thinking (NCTM, 1989), rather than just as the means to an end of finding the correct answer. Student exploration may also promote a stance of mathematics as a discipline that can be explored, reasoned about, connected to other subjects, and one that 'makes sense' (Barker, et al., 2004).

> Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

If students are following a procedure established by the teacher, then it does not count as exploration/investigation/problem solving. Instead, students should be determining their own solution pathway without necessarily knowing that the path will lead to the desired result.

Score	Description
3	Students regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the students engaged in exploration/investigation/problem solving.
2	Students sometimes engaged in exploration, investigation, or problem solving. Several students engaged in problem solving, but not the majority of the class.
1	Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or a few students engaged in problem solving while other students watched but did not actively participate.
0	Students did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving, or the instances were carried out by the teacher without active participation by any students.

2) **Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts.**

In mathematics instruction it is common for the teacher to use various representations (models, drawings, graphs, concrete materials, manipulatives, graphing calculators, compass & protractor, i.e. tools for the mathematics classroom) to focus students' thinking on and develop their conceptions of a mathematical concept. It is also important for students to interact with and develop representations of mathematical concepts and not merely observe the teacher presenting such representations. Thus, this item is concerned with whether the students use representations to represent mathematical concepts. The representations can be student generated (a drawing or a graph) or provided by the teacher (manipulatives or a table), but it is the students that must then use the representation. Just because there is a representation in a lesson, if it is only used by the teacher while students watch (such as a graph on a PowerPoint slide), it is not considered to be used by students unless the students manipulate and interact with the representation.

Students' notes can count as a type of representation if the students themselves offer some sort of input. For instance, if a student corrects a teacher's mistake in a problem he or she is copying down then the notes are actually being manipulated by a student and should therefore count as a type of representation.

Score	Description
3	The students manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or students, as appropriate.
2	The students manipulated or generated two or more representations to represent the same concept, but the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were **not** explicitly discussed by the teacher or students.
1	The students manipulated or generated one representation of a concept.
0	There were either no representations included in the lesson, or representations were included but were exclusively manipulated and used by the teacher. If the students only watched the teacher manipulate the representation and did not interact with a representation themselves, it should be scored a 0.

3) Students were engaged in mathematical activities.

This item is concerned with the extent of student engagement in activities that are mathematical. Students are considered to be engaged in a mathematical activity when they are investigating, problem solving, reasoning, modeling, calculating, or justifying (each of these could be written or verbal).

Note “most of the students” in an undergraduate mathematics classroom is accepted here to mean more than one-third of the students in the classroom were engaged in mathematical activity, while in a K-12 mathematics classroom it means more than one-half.

It is important to note that one should only focus on what actually happens—not what the teacher assigns watching for students who are off-task.

Score	Description
3	Most of the students spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the students are filling in the notes and interacting with the lesson mathematically.)
2	Most of the students spend more than one-quarter but less than two-thirds of the lesson engaged in appropriate level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.
1	Most of the students spend less than one-quarter of the lesson engaged in appropriate level mathematical activity. There is at least one instance of students’ mathematical engagement.
0	Most of the students are not engaged in appropriate level mathematical activity. This could be because they are never asked to engage in any activity and spend the lesson listening to the teacher and/or copying notes, or it could be because the activity they are engaged in is not mathematical – such as a coloring activity.

4) **Students critically assessed mathematical strategies.**

In order for students to flexibly use mathematical strategies, they must develop ways to consider the appropriateness of a strategy for a given problem, task, or situation. This is because not all strategies will work on all problems, and furthermore the efficiency of the strategy for the given context needs to be considered. For students to make such distinctions it is important that they have opportunities to assess mathematical strategies so that they learn to reason not only about content but also about process. This item is concerned with *students* critically assessing strategies, which is more than listening to the teacher critically assessing strategies or asking peers how they solved a task. Examples of critical assessment include students offering a more efficient strategy, asking "why" a strategy was used, comparing/contrasting multiple strategies, discussing the generalizability of a strategy, or discussing the efficiency of different ways of solving a problem (e.g. the selection appropriate tools if needed).

To score high on this item it is the students who must be engaged in the critical assessment, not only the teacher.

Score	Description
3	More than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
2	At least two but less than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
1	An individual student critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher. The critical assessment was limited to one student.
0	Students did not critically assess mathematical strategies. This could happen for one of three reasons: 1) No strategies were used during the lesson; 2) Strategies were used but were not discussed critically. For example, the strategy may have been discussed in terms of how it was used on the specific problem, but its use was not discussed more generally; 3) Strategies were discussed critically by the teacher but this amounted to the teacher telling the students about the strategy(ies), and students did not actively participate.

5) Students persevered in problem solving.

One of the *Standards for Mathematical Practice* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) is that students will persevere in problem solving. Student perseverance in problem solving is also addressed in the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics Curriculum Guide (Barker, et al., 2004):

> Every course should incorporate activities that will help all students…approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures.

Perseverance is more than just completion or compliance for an assignment. It should involve students overcoming a road block in the problem solving process.

Score	Description
3	Students exhibited a strong amount of perseverance in problem solving. The majority of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
2	Students exhibited some perseverance in problem solving. Half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
1	Students exhibited minimal perseverance in problem solving. At least one student but less than half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), at least one student but less than half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem. There must be a road block to score above a 0.
0	Students did not persevere in problem solving. This could be because there was no student problem solving in the lesson, or because when presented with a problem solving situation no students persevered. That is to say, all students either could not figure out how to get started on a problem, or when they confronted an obstacle in their strategy they stopped working.

6) **The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.**

Relational/conceptual understanding is “knowing both what to do and why” (Skemp, 1976). This is in contrast to a procedural understanding as being able to compute certain mathematical activities, but not understanding how the computation works or when one would need to use such a computation and what the answer would mean.

According to th**e** NCTM (2006), certain topics are core to the mathematics learned at each grade level and can form the backbone of the K-8 curriculum. The NCTM extended this concept to the high school level with an emphasis on using these fundamental concepts to make sense of mathematics and deepen students’ relational and conceptual understanding (Martin, et al., 2009). Similar to the NCTM’s guidelines for middle school and high school mathematics lessons, at the undergraduate level the Mathematical Association of America has recommendations in the Committee on the Undergraduate Program in Mathematics Curriculum Guide (Barker, et al., 2004) for departments, programs, and all courses to promote relational/conceptual understanding for both mathematics majors and non-mathematics majors.

Score	Description
3	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
2	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, but the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
1	The lesson mentions some fundamental concepts of mathematics, but does not use these concepts to develop the relational/conceptual understanding of the students. For example, in a lesson on the slope of the line, the teacher mentions that it is related to ratios, but does not help the students to understand how it is related and how that can help them to better understand the concept of slope.
0	The lesson consists of several mathematical problems with no guidance to make connections with any of the fundamental mathematical concepts. This usually occurs with a teacher focusing on procedure of solving certain types of problems without the students understanding the “why” behind the procedures.

7) **The lesson promoted modeling with mathematics.**

Following the "Standards for Mathematical Practice" from the Common Core State Standards (2010) and the recommendations from the MAA's CUPM Curriculum Guide (Barker, et al., 2004), this item describes lessons that help students to "apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

In an undergraduate classroom, a lesson that promotes modeling might use "radiocarbon dating to illustrate how an initial value problem (IVP) can model a real world situation, and the solution of the IVP then yields obviously useful and interesting results" or "a simple system of differential equations to predict the cyclical population swings in a predator-prey relationship" or even "how modular arithmetic is used in cryptography and the transmission of encoded information" (Barker, et al., 2004).

Score	Description
3	Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson with students engaged in the modeling cycle (as described in the Common Core State Standards).
2	Modeling is a major component, but the modeling has been turned into a procedure (i.e. a group of word problems that all follow the same form and the teacher has guided the students to find the key pieces of information and how to plug them into a procedure.); **<u>or</u>** modeling is not a major component, but the students engage in a modeling activity that fits within the corresponding standard of mathematical practice.
1	The teacher describes some type of mathematical model to describe real-world situations, but the students do not engage in activities related to using mathematical models.
0	The lesson does not include any modeling with mathematics.

8) The lesson provided opportunities to examine mathematical structure. (Symbolic notation, patterns, generalizations, conjectures, etc.)

Following some of the "Standards for Mathematical Practice" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) and the recommendations in the MAA's CUPM Curriculum Guide (Barker, et al., 2004), lessons should include opportunities for students to contextualize and/or decontextualize in the process of solving quantitative problems, explore and make use of mathematical structure, or to use repeated reasoning to generalize certain categories of problems and their solutions.

Score	Description
3	The students have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns.
2	Students are given some time to examine mathematical structure, but are not allowed adequate time or are given too much scaffolding so that they cannot fully understand the generalization.
1	Students are shown generalizations involving mathematical structure, but have little opportunity to discover these generalizations themselves or adequate time to understand the generalization.
0	Students are given no opportunities to explore or understand the mathematical structure of a situation.

9) **The lesson included tasks that have multiple paths to a solution or multiple solutions.**

As part of having students "make sense of problems and persevere in solving them" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), students must be encouraged to look for multiple methods of solving a problem and to deal with problems that have multiple solutions based upon various assumptions. Additionally, selected tasks with multiple paths to a solution or multiple solutions can increase the cognitive demand of the task for all students through the interaction of the teacher to ask questions of each student at their ability level (Stein & Smith, 1998). This flexibility, "switching (smoothly) between different strategies," and adaptivity, "selecting the most appropriate strategy" (Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009) enables students to solve problems for which a solution path is not obvious.

Score	Description
3	A lesson which includes several tasks throughout; or a single task that takes up a large portion of the lesson; with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different students.
2	Multiple solutions and/or multiple paths to a solution are a significant part of the lesson, but are not the primary focus, or are not explicitly encouraged; **or** more than one task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
1	Multiple solutions and/or multiple paths minimally occur, and are not explicitly encouraged; **or** a single task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
0	A lesson which focuses on a single procedure to solve certain types of problems and/or strongly discourages students from trying different techniques.

10) The lesson promoted precision of mathematical language.

This item follows the Standard of Mathematical Practice to “attend to precision”. As such, “Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem” (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

This item also follows the MAA’s CUPM Curriculum Guide recommendation to “develop mathematical thinking and communication skills” which states: “Students should read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking” (Barker, et al., 2004).

Whether the communication is verbal or written and originating in the teacher or a student, using precise mathematical language is important. While the teacher cannot control the language used by students, there should be evidence of expectations of the teacher upon the students related to communicating with precise mathematical language. For example, if the lesson is primarily students solving problems, a culture of precision of language should come through in how the students are communicating with one another, both verbal and written.

Score	Description
3	The teacher “attends to precision” in regards to communication during the lesson. The students also “attend to precision” in communication, or the teacher guides students to modify or adapt non-precise communication to improve precision.
2	The teachers “attends to precision” in all communication during the lesson, but the students are not always required to also do so.
1	The teacher makes a few incorrect statements or is sloppy about mathematical language, but generally uses correct mathematical terms.
0	The teacher makes repeated incorrect statements or incorrect names for mathematical objects instead of their accepted mathematical names.

11) The teacher's talk encouraged student thinking.

This item assesses how well the teacher's talk promotes a number of the mathematical practices. Specifically, the practices requiring students to be able to think, reason, argue, and critique during the study of mathematical concepts (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Teachers can greatly impact the level of student thinking and discussion simply by what questions are asked of students. In line with Stein, et al. (2009), the cognitive task level should be maintained at a high level, i.e. procedures with connections and doing mathematics, while questions which are over-scaffolded, rhetorical, or cursory to the level of the students, would score a 1 or a 0.

Specifically about the teacher's talk, this item is referring to the content of the question or statements put forth in the classroom for students to reason and/or discuss. A well planned lesson may contain rich tasks for students to explore or problems to solve, but if the teacher's talk drops or removes student reasoning and problem solving, it has removed or reduced student thinking.

Score	Description
3	The teacher's talk focused on **high levels** of mathematical thinking. The teacher may ask lower level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. ***Analysis****:* examines/ interprets the pattern, order or relationship of the mathematics; parts of the form of thinking. **Synthesis**: requires original, creative thinking. **Evaluation**: makes a judgment of good or bad, right or wrong, according to the standards he/she values.
2	The teacher's talk focused on **mid-levels** of mathematical thinking. **Interpretation**: discovers relationships among facts, generalizations, definitions, values and skills. **Application**: requires identification and selection and use of appropriate generalizations and skills
1	Teacher talk consists of **"lower order"** knowledge based questions and responses focusing on recall of facts. **Memory**: recalls or memorizes information. **Translation**: changes information into a different symbolic form or situation.
0	Any questions/ responses of the teacher related to mathematical ideas were rhetorical in that there was no expectation of a response from the students.

12) There were a high proportion of students talking related to mathematics.

The focus of this descriptor is on the proportion of students talking (frequency). The Standards for Mathematical Practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) encourages students to be active in making conjectures, exploring the truth of those conjectures, and responding to the conjectures and reasoning of others. In a classroom dominated by only a few students, classroom discourse may appear to be high, but all students must be engaged.

Score	Description
3	More than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
2	More than half, but less than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
1	Less than half of the students were talking related to the mathematics of the lesson.
0	No students talked related to the mathematics of the lesson.

13) There was a climate of respect for what others had to say.

This item adheres to the expectation provided in the third Standard for Mathematical Practice, "Construct viable arguments and critique the reasoning of others." Given that practice, students are expected to communicate with each other as part of an effective classroom community. Effective communication means that students will listen, question, and critique; this is part of the discourse expected in a mathematics classroom (Sherin, Mendez, & Louis, 2004). This item also encompasses the literature on equity and mathematics in that all students have valuable ideas, strategies, and thinking to share within the mathematics classroom (Boaler, 2006). Equitable spaces include the interactions of students within a mathematical community that increase participation and engagement of all students and work to remove potential barriers (Diversity in Mathematics Education Center for Learning and Teaching, 2007; Gutierrez, 2007; Hiebert & Grouws, 2007; NCTM, 2000; Sherin, Mendez, & Louis, 2004; Yackel & Cobb, 1996). This means creating a climate of respect.

Score	Description
3	**Many** students are sharing, questioning, and commenting during the lesson, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others.
2	The environment is such that **some** students are sharing, questioning, and commenting during the lesson, including their struggles. Most students listen.
1	Only a **few** share as called on by the teacher. The climate supports those who understand or who behave appropriately. **Or** Some students are sharing, questioning, or commenting during the lesson, but most students are actively listening to the communication.
0	No students shared ideas.

14) In general, the teacher provided wait-time.

The appropriate wait time must align with the question/task. In the elementary grades, a teacher may ask students to explain a situation that represents the expression 24*(1/2)*3. In middle school, the teacher may ask students to describe why the slope is positive. High school teachers may ask students to explain how linear and exponential functions are similar and different. In each instance, these questions/tasks are not simple yes/no answer and require wait time to provide an answer with meaning and understanding.

Simple Yes/No questions could be asked, but must be accompanied by an explanation. Simple skills or procedural problems should require explanations with the computation and/or procedures. If the class is dominated by rhetorical questions, a score of 0 or 1 is warranted. Even if rhetorical questions are asked, it is possible to score a 2 or 3 if there are questions asked sometimes or frequently that require students to reason, make sense, and articulate thoughtful responses.

Score	Description
3	The teacher **frequently** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
2	The teacher **sometimes** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
1	The teacher **rarely** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
0	The teacher **never** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.

15) Students were involved in the communication of their ideas to others (peer-to-peer).

Both the National Council of Teachers of Mathematics and The Eight Standards for Mathematical Practices, expect teachers to create a mathematical community that includes dialogue around the mathematics content and learning. Students are expected to talk and participate in the discourse of the classroom (Manouchehri & St John, 2006). This item highlights the need for all students to be active participants in the classroom dialogue. Without teacher support and expectations, the classroom discourse can be monopolized or biased against certain populations (Mercer & Wegerif, 1999; Mercer, Wegerif, & Dawes, 1999; Rojas-Drummond & Mercer, 2003; Rojas-Drummond & Zapata, 2004).

This descriptor focuses on the amount of time students spend in communication with their peers at any level, including pairs, groups, informal settings, or whole class settings.

Score	Description
3	**Considerable** time **(more than half)** was spent with peer to peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies and solution.
2	**Some** class time **(less than half, but more than just a few minutes)** was devoted to peer to peer (pairs, groups, whole class) conversations related to the mathematics.
1	The lesson was primarily teacher directed and little opportunities were available for peer to peer (pairs, groups, whole class) conversations. A few instances developed where this occurred during the lesson but only lasted less than 5 minutes.
0	No peer to peer (pairs, groups, whole class) conversations occurred during the lesson.

16) **The teacher uses student questions/comments to enhance conceptual mathematical understanding.**

Driscoll (1999; 2007) and Reys, et al. (2009) discuss how teacher questioning can build on student thinking to foster deeper mathematical thinking. In the elementary grades, students can make “over generalized” statements that have a correct nature about them. This is a teachable moment to use. A teacher can ask a question that has the student(s) reexamine their thoughts that would help simplify the over generalizing statement into precise understanding. Reys, et al. (2009) present a simple example, “Student: So every even number is composite. Teacher: Every even number? <Pause with wait time> What about 2?” The teacher’s question stimulates further thought by the student. In secondary grades, Driscoll (1999) indicates that well-timed questions to students should help them shift or expand their thinking, or at least have students thinking about what is important to pay attention to during a lesson. When students are examining expressions, a teacher can ask questions to facilitate mathematical flexibility (Heinze, Star, & Verschaffel, 2009). For example, “What other ways can you write that expression to bring out the hidden meaning? How can you write the expression in terms of the important things you care about?”

Score	Description
3	The teacher **frequently** uses student questions/ comments to coach students, to facilitate conceptual understanding, and boost the conversation. The teacher sequences the student responses that will be displayed in an intentional order, and/or connects different students’ responses to key mathematical ideas.
2	The teacher **sometimes** uses student questions/ comments to enhance conceptual understanding.
1	The teacher **rarely** uses student questions/ comments to enhance conceptual mathematical understanding. The focus is more on procedural knowledge of the task verses conceptual knowledge of the content.
0	The teacher **never** uses student questions/ comments to enhance conceptual mathematical understanding.

Equitable Mathematics Teaching Practices

As we consider and observe the teaching and learning of mathematics, we have to consider if the experience is equitable for each and every student. This means that every student has the opportunity to engage in a rigorous, authentic, and relevant mathematics experiences in order to prepare them for a successful future (Stinson, 2004). Historically, many practices observed in mathematics classrooms have resulted in dehumanizing students of color (Delpit, 2006) and students with disabilities (Lambert, & Stylianou, 2013; i.e., tracking, procedural tasks, skill and drill).

Equitable teaching ensures that all students are able to have entry into the mathematics experience, construct their own knowledge, and share their reasoning, strategies, and solutions with others thus creating a positive classroom community (Boaler & Greeno, 2000; Boaler, 2005, 2009; Gutstein & Peterson, 2005). Practices to observe would be teachers relating to students' gender, class, race, and culture through authentic connections (Aguirre, Mayfield-Ingram, & Martin, 2013; Aguirre & Zavala, 2013; Ladson-Billings, 1995). When observing mathematics teaching, one should consider if all students have power and are active participants during the learning episode, this means that all are worthy of sharing, speculating, reasoning, and processing with their peers. The practice of teaching and learning mathematics is identity work (Martin 2000; Gutiérrez 2013), and we must be cognizant of practices that attack the identities of students and prevent them from succeeding.

***Disclaimer:** This does not mean that differentiation may not be warranted. Equitable teaching with differentiation would mean the differentiated experience was still valuable and connected to the learning of their peers and not an isolated skill and drill session or procedural worksheets in the place of investigations, problem solving, and/ or inquiry. These procedural types of activities would occur during Response to Intervention (RTI) time slots and not encapsulate the normal mathematics instruction.

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Appendix of Operationalized Terms

Problem Solving (PS) – Items 1 & 5 – for the purposes of using the MCOP^2, users should consider whether problems that appear during the observed lesson, appear well-selected and meaningful to the mathematics of the lesson. Problems should not be short repeated exercises of a skill and/or procedure only, as these are exercises and not problems. Problems would include characteristics that require students to think, make sense, struggle at times, and encounter some hurdle, shift, or change (roadblock) that may require using tools or adjusting a prior method from earlier in the lesson or unit. [*We refer readers to two specific readings on problem solving, the NCTM (2000) Principles and Standards for School Mathematics and the Principles to Actions (NCTM, 2014).*]

Specifically, item 5 is to capture problem solving with perseverance with overcoming roadblocks through problem solving as the underlying aspect and does not include exploration and/or investigation. Item 5 scores a zero if a lesson is fully exploration and/or investigation without any problem solving. An observer must examine if struggles are overcome by teacher direction (too much scaffolding) or if students overcome struggles using resources which may include brief commentary from a teacher without taking away the struggle(s). The brief commentary may, for example, be students conferring with the teacher without taking away the struggle but instead may use questioning strategies that may spark or open up the struggle to a new route of problem solving or reasoning. A roadblock is defined as something new or unique to a problem for which students have not seen yet during the lesson observed and/or a point of problem solving in which the observer can see and/or hear students struggling and/or trying to make sense of the situation.

Item 1 may score identically if a lesson includes enough problem solving and perseverance by the majority of students in the class. Whereas, a lesson with extensive exploration and investigation that does not include problem solving would score well on item 1 but not item 5. Conversely, a lesson observed that is primarily focused on problem solving, may score well on item 5 with observed perseverance by many students and in turn would score well on item 1. It is not foreseen that item 5 scores well and item 1 scores poorly.

Mathematical Activity – Item 3 – An activity should be considering anything students do that is mathematical in nature as seen in the rubrics on an appropriate level. Meaning, students in Algebra 1 being asked to do computation problems regarding numeric fractions, would not be considered an appropriate level activity. Whereas, being asked to do this computation work in 5th or 6th grade would likely be an appropriate level activity. Similarly, this item would have a low score if the students are primarily engaged in non-mathematical activities such as coloring or crafting related to the mathematics lesson. Item 3 would score high if items 1 & 5 scored high, whereas item 3 can score high but not necessarily other items related to what students do. A lesson that introduces a new topic might involve a majority of time taking notes, Q&A, but not involve problem solving or exploration. Students may complete some basic exercises during the lesson. Given item 3 would score high, many other items would likely not score high in this later situation.

Mathematical Strategy – Item 4 – for the purposes of using the MCOP^2, users should consider strategies in a number of situations. Strategies apply to all aspects of being able to move forward productively during a teacher or student created situation, problem, or task. We define strategy as a potential pathway for making progress in the observed classroom situation. That is, strategies can be a number of different aspects seen/heard during a lesson. Moreover, this item loads on both the teacher facilitation and student engagement factors of measure on the MCOP^2. Therefore, it is imperative for teachers to put students in situations to be able to be critical of the situation, problem, or task. But, students themselves must be critical given the opportunity. For example, students may be asked to assess other students' work. In this situation, students must demonstrate a critical assessment or interpretation of their peers' work and provide an explanation of their assessment. Another example may include students recognizing a mistake or struggle to complete a task in groups, thus they question whether or not they should continue or change direction. A teacher may ask a good question or present a possible conclusion, while students are critical about the question or statement. Students may ask why or how about the situation, task, or problem at-hand based on aspects of the lesson that is ongoing. Strategies may be procedures or representations used during problem solving, as well as students' process to move forward during exploration or investigation as it relates to other items, for example like previous described items 1 & 5.

Modeling – Item 7 – Caution: This has long been a confused item. As intended on this item, modeling is when students use mathematical models to describe/solve/understand real world situations and problems as described in the GAIMME report (2016).

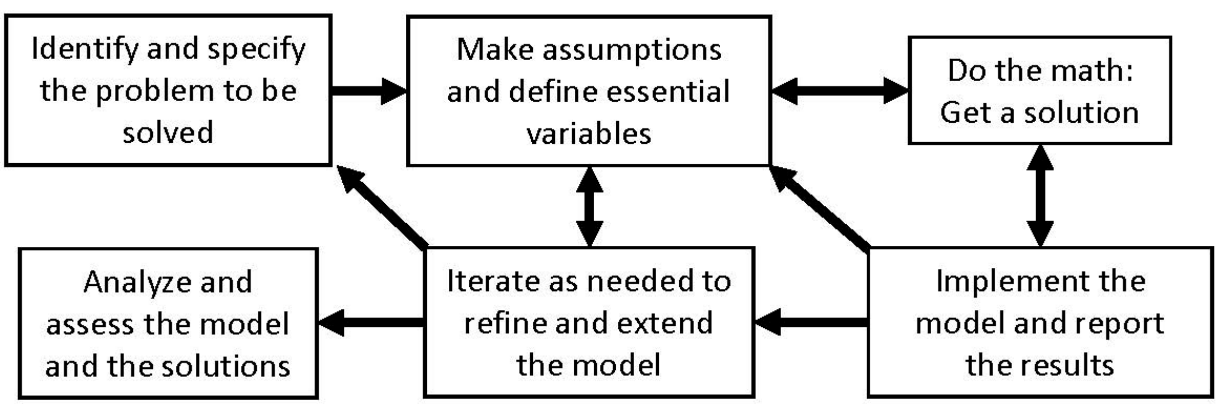

Figure 1: Representation of the Modeling Cycle (GAIMME 2016, p. 31)

Students need not fully complete the modeling cycle during the lesson, but be engaged in one or more parts of the modeling cycle (see Figure 1).

This definition of modeling is in contrast to using manipulatives, equations, pictures, graphs, multiple representations, etc. to model mathematical concepts and ideas. For example, students may use pictures or concrete manipulatives to make sense of two-digit multiplication or completing the square of a trinomial. While these models can be very effective in learning and understanding procedures and concepts during instruction, they are models of mathematical concepts or situations. <u>***Without a real world situation, context, or problem***</u> in which students are trying to make sense or solve a problem, these uses of manipulatives, etc, is modeling a mathematical concept to deepen understanding but not modeling as expected in the modeling aspects of the Mathematics Practice Standard or the GAIMME report.

<u>Climate/Environment</u> – Item 13 – For users of the MCOP^2, we operationalize that these two terms are one in the same. This item was found to load onto both measurable factors of the MCOP^2, teacher facilitation and student engagement. The research was clear, it is a responsibility of both students and the teacher to create an environment/climate in which students can learn. Observers should be internally asking and looking for...

Does the teacher make valid and equitable attempts to give students opportunities to participate?
Does the teacher recognize and address situations moving in a direction that may be negative?
Do students participate frequently when asked questions or given problems or tasks?
Do students help steer group members off-task or struggling?
Does the teacher make an attempt (1-1, in groups, or whole class) to involve many students?
Does the climate promote positive mathematical identities for all students?

The <u>climate/environment</u> are all classroom aspects an observer may see and/or hear that relate to students being able to engage and learn as setup or facilitated by a teacher. Of special note, some days some students choose for good reasons, to remain silent and listen, or in some instances, just space out or put their head down. Good teachers may well already know without providing an observer information needed. An observer should make notes of such situations and debrief with a teacher before making a final judgment. Many students may be considered the majority in larger classes, whereas in small classes, it might be expected to have a higher threshold than just the majority. A lesson in which the teacher expects students to sit quietly, take notes, and do some exercises, very likely would be a climate that is scores no higher than a 1. One should also consider the “closeness” for scores of 1&2. The “or” statement in a score of 1, is extremely close to the score of 2. However, an observer must be able to see and/or hear students including their struggles during the lesson to warrant a 2. In a classroom environment that is safe enough for students to share struggles and feel comfortable enough to do so, we have an environment that improves opportunity for students to learn.

<u>Precision</u> – Item 10 – For users of the MCOP^2, one must consider precision as defined in the mathematical practice standards. Moreover, we further define precision for this item. When scoring this item, <u>precision</u> should mean the intent of teachers to make valid attempts to have precise communication during the lesson and should not be a "gotcha" type scoring. A teacher may make a small incorrect use of a word once or twice (e.g. expression↔equation, fraction↔ratio) during a lesson that does not impact the quality of the lesson or increase student confusion, this would score a 2 unless there are clear misconceptions that may follow as a result. For example, when teaching about transformations the teacher may use the word "move" some of the time and "translate" at other times to reference the same mathematical idea. But, when this observation becomes obvious after a few times or the teacher uses very slang mathematical terms (or incorrect language such as "alligator mouths" when referencing inequalities or "put f and g together" in reference to composition of functions) to the observer, then there is a situational issue and we see a differentiation between a score of a 1 and a 2. To score a 3, a teacher must help students see their language is not valid or accurate and prompt for restatement or revision of verbal or written communication. This item is solely about teacher responsibility and communication during the lesson, so while observers may focus on what students say, the only instance that impacts the observational scoring of this item is what teachers do (or not).

<u>Wait time vs think time</u> – Item 14 – For users of the MCOP^2, we operationalize and differentiate these two terms. <u>Think time</u> is an appropriate amount of time for students to think about the question or situation being presented. This item is specifically about whether or not students have enough time to think about what they are confronted with at the moment. While the item states "the teacher provided", based on the validity research during development of the MCOP^2, this item is specifically about whether or not students have ample think time. One should not score the item based on "teacher" aspects of <u>wait time</u> since teachers may need a follow-up question quickly if a student shows signs quickly of not understanding the question or task. Rather, as stated in the item rubric, whether or not the observer witnesses students having think time about the actual situation during the lesson is the look-for for observers. One may consider "rarely" depending on the length of the class period once or twice during a 50-minute lesson where a few times might be rarely for a 90-minute block.